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On Modeling U.S. Product Liability Risk
- An Empirical Analysis -

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On Modeling U.S. Product Liability Risk
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Abstract

The purpose of this paper is to seek a model that adequately describes a company's U.S. product liability risk based on a company's loss data. U.S. product liability risks are real threats to Japanese multinational corporations which sell their products to the U.S. market since the liability risks might cost a tremendous amount of money to them not only for the liability claims but also for punitive damages, legal costs, reputational costs, business opportunity costs and risk control costs. Thus, it is important for the corporations to manage the U.S. product liability risk effectively and efficiently. The models can be used by the corporate risk managers for the insurance cost and benefit analysis, the captive feasibility study, the risk management efficiency analysis, among others.

Based on the actual U.S. product liability loss data of a particular manufacturer [1], five different compound Poisson process models, which are Poisson processes compounding with a jump following Inverse Gaussian Distribution, Lognormal Distribution, Gamma Distribution, Pareto Distribution or Weibull Distribution, are analyzed for their goodness-of-fit to the risk. The parameters for each model are estimated by the maximum likelihood estimation method. The best fitted model is determined based on AIC criterion.

As a result, the Lognormal type compound Poisson process is the best among the models to describe the risk. Kolmogorov-Smirnov test confirms the result that the model is “ not significantly ” different from the actual process.

Key words: U.S. Product Liability, Risk Management, Risk Modeling, Compound Poisson Process,

1 Introduction

Japanese multinational corporations, especially manufacturers who export their products to foreign markets, face various international risks. One of their serious concerns in exporting goods to foreign countries is the exposure[2] to the country's civil liability for defects in the products, namely, product liability risks. Especially, the product liability risk in the U.S. is considered a real threat to the Japanese manufacturers since the liability, when it incurs, often costs the companies a tremendous amount of money not only for liability claims themselves but also for additional costs such as punitive damages, legal costs, reputational costs, business opportunity costs and risk control costs.

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[1]called ”Company A” in this paper.

[2]International Risk Management Institute [5] defines ”exposure” as the state of being subject to loss because of hazard or contingency. Also used as a measure of the rating units or the premium based on a risk.

This paper presents an empirical analysis to show how the companies can model the risk assuming that it follows the Lévy process on the U.S. product liability losses. The paper, therefore, focuses on the analysis of the actual U.S. product liability loss data and then examines which stochastic model the most appropriately describes the risk.

The structure of the subsequent sections is as follows: In the second section, product liability in the U.S. is briefly described. The section discusses what the U.S. product liability risk is all about and the necessity of the research. The third section demonstrates the background literature and discusses what models can be the candidate models to the risk. The fourth section explains the nature of the loss data used for modeling. The fifth section describes the focused models in more details. We focused several Lévy processes for modeling. They are compound Poisson processes with Lévy measures such as Inverse Gaussian, Lognormal, Gamma, Pareto and Weibull distributions. The sixth section presents the methodology to the parameter estimation for the models and their comparison of their goodness-of-fit to the risk. Here, the classical method of moments is used to provide initial input values for parameters. The maximum likelihood estimation method is then used to estimate model parameters. The candidate models are compared based on AIC (Akaike Information Criterion) to determine the best fitted model among them. The seventh section discusses the results, implication and limitations of this research. The last section is left for concluding remarks.

2 Product Liability

Metzger et al. [6] defines ”Products Liability” as the legal responsibility of the manufacturer, distributor, or retailer to the user or consumer of a product. The liability arises out of the manufacture, distribution, or sale of an unsafe, dangerous, or defective product and the failure of the manufacturer, distributor, or retailor to meet the legal duties imposed with respect to the particular product.

They[6] also argues that most products liability lawsuits in the U.S. are based on negligence or strict liability in tort or both. In these days, the law is at the consumer side and is more likely to protect consumers. Modern courts and legislatures intervene in private contracts for sale of goods and impose liability regardless of fault. As a result, sellers and manufacturers face greater liability and higher damage recoveries for defects in their products. Japanese manufactuers who are unfamiliar with the U.S. legal systems face more exposure to the liability risk since they are less likely prepared for the risk than the U.S. companies.

Japanese companies often became targets for injured people together with their attorneys seeking for larger monetary recoveries and compensations because they know that Japanese companies are vulnerable. Therefore, it is of great value if the Japanese companies could forecast future losses based on the risk model for the risk management purposes. Cost benefit analysis for insurance purchase and captive feasibility study are some examples for its usage.

In order to minimize the cost associated with the U.S. product liability risk, Company A might want to establish a captive insurance company, its insurance subsidiary company, to obtain adequate coverage for the U.S. product liability risk rather than to purchase an expensive product liability insurance from the commercial insurance market. Skipper et al. [2] defines ”Captives” as closely held insurance companies that primarily underwrite the risks of their owners. They argue that captives can provide corporations various benefits such as reduced costs, access to reinsurance, cash flow advantages, investment income and tax advantages.

Confirming the feasibility of its captive, Company A should conduct a feasibility study to determine whether or not and to what extent the captive can maintain its solvency. Initial capital investment and the premium are two of the most important determinants in the feasibility study.

They can be numerically determined in the simulations of proforma financial statements which should include forecasted losses. The future losses can be forecasted with a stochastic model that is created based on the past loss data. This paper, therefore, attempts to determine an appropriate model to be used for the feasibility study assuming that Company A wants to create a captive to cover the U.S. product liability risk. The ultimate concern is to obtain the distribution of losses at each year end over the forecasted time period so that the initial capital and premiums should be enough to cover the loss distribution at a certain confidence level.

3 Background Literature and Candidate Models

The accumulated losses follows the aggregate claims process. Gerber [3] argues that a compound Poisson process with stationary and independent increments can be appropriate for the aggregate claims process. Here, the claim number process follows Poisson process. The increments are considered to follow a certain loss distribution or mixed loss distributions.

As far as loss distributions are concerned, Hogg and Klugman [4] suggest that for *Size-of-Loss Distributions*, Pareto Distribution, Gamma Distribution, Lognormal Distribution and Weibull Distribution can be the candidate.

This paper focused on the following five distinct distributions: Inverse Gaussian Distribution, Gamma Distribution, Lognormal Distribution, Pareto Distribution and Weibull Distribution.

4 Data and Sample Path

The data initially collected was limited to the product liability losses incurred during the year 1980-1996 to Company A, which exports products to the U.S. market.

Company A started exporting its products to the U.S. market in the late 1970s. The company first suffered from product liability losses in 1980 and, since then, the number of losses has been increased as its U.S. sales increased until the 1990s when the annual loss amount has been rather stablized.

For developing risk models, the last five year loss data which are the data from 1992 to 1996 are used as the sample data because those five years are assumed to reflect the most current business conditions and also in these years the U.S. sales figure has become stable. Trend, loss developing and incremental exposure are ignored for simplicity in this analysis. The sample data for this research is illustrated in the next page.

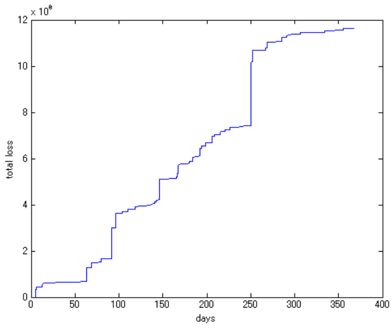


Fig. 1 Year 1992

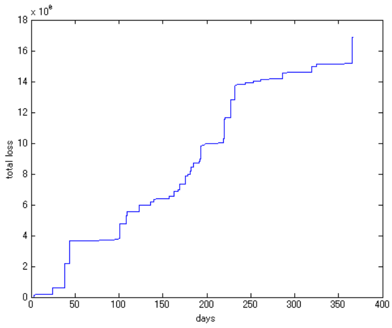


Fig. 2 Year 1993

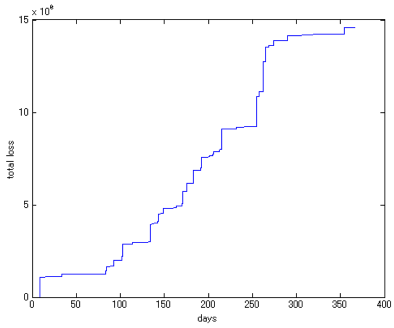


Fig. 3 Year 1994

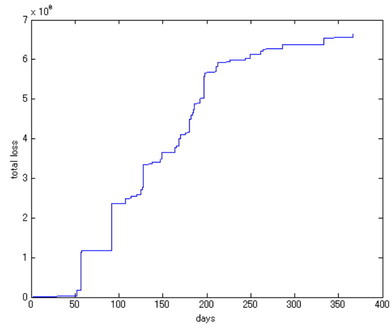


Fig. 4 Year 1995

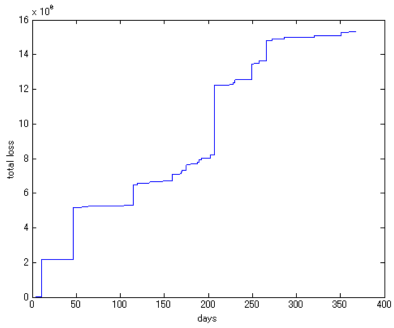


Fig. 5 Year 1996

5 Model Description

Lévy Process $\{Z_t\}$ follows a compound Poisson process when it has generating triplets of the following: $(0, \nu(dx), b_0)_0$, $\nu(dx) = c\rho(dx)$. c is a positive number and $\rho(dx)$ is a probability measure on $\mathbb{R}$ where $\rho(\{0\}) = 0$. c is a parameter that indicates a frequency of Jump occurrence and $\rho(dx)$ follows a distribution of jumps if a jump occurred.

In this case, Z_1 has a characteristic function of $\phi(u)$:

$$\begin{aligned}
\phi(u) &= \exp[\psi(u)] \\
\psi(u) &= ib_0 u + c(\hat{\rho}(u) - 1) \\
\hat{\rho}(u) &= \int_{\infty}^{\infty} e^{iux} \rho(dx)
\end{aligned}$$

In this study, the following mathematical models are focused and analyzed for their fitness to the product liability risk:

1. IG type compound Poisson process: the Lévy measure is the Inverse Gaussian (IG) distribution
2. Lognormal type compound Poisson process: the Lévy measure is the Lognormal distribution.
3. Gamma type compound Poisson process: the Lévy measure is the Gamma distribution.
4. Pareto type compound Poisson process: the Lévy measure is the Pareto distribution.
5. Weibull type compound Poisson process: the Lévy measure is the Weibull distribution.

Each model is briefly explained in this section. Here, unless otherwise noted, the following equations are satisfied:

$$\begin{aligned}
\hat{h}_1 &:= \hat{m}_1 \\
\hat{h}_2 &:= \hat{m}_2 - \hat{m}_1{}^2 \\
\hat{h}_3 &:= \hat{m}_3 - 3\hat{m}_2\hat{m}_1 + 2\hat{m}_1{}^3 \\
\hat{m}_k &:= \frac{1}{n}\sum_{i=1}^{n} \xi_i{}^k,\ k = 1, 2, 3
\end{aligned}$$

5.1 Inverse Gaussian (IG) Type Compound Poisson Process

IG distribution ρ is given by

$$\rho(B) = \frac{\alpha}{\sqrt{2\pi}} \exp(\alpha\beta) \int_B x^{-3/2} \exp\left(-\frac{1}{2}\left(\alpha^2 x^{-1} + \beta^2 x\right)\right) 1_{\{x>0\}}\, dx$$

It is known that the distribution of the following stopping time, $T_{(\alpha,\infty)}$,

$$T_{(\alpha,\infty)} = \inf\{t > 0 : \beta t + W_t > \alpha, \alpha > 0, \beta > 0\}$$

follows the IG distribution. Here, $\{W_t\}$ is a Brownian Motion.

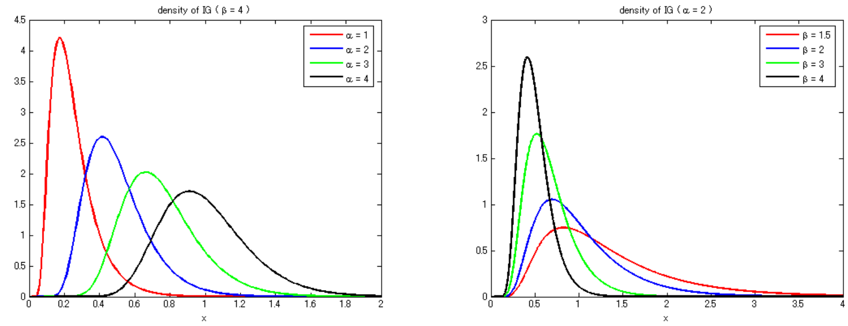


Fig. 6 Density Function of IG Distribution

The characteristic function, $\hat{\rho}(u)$, is described as :

$$\hat{\rho}(u) = \exp\left[-\alpha\left(\sqrt{-2iu+\beta^2}-\beta\right)\right]$$

Therefore, Lévy Process, $\{Z_t\}$, that follows the IG type compound Poisson process whose Lévy measure is :

$$\nu(dx) = c\frac{\alpha}{\sqrt{2\pi}}\exp(\alpha\beta)x^{-3/2}\exp\left(-\frac{1}{2}\left(\alpha^2x^{-1}+\beta^2x\right)\right)1_{\{x>0\}}\,dx$$

In this case,

$$\begin{aligned}\phi(u) &= \exp\{\psi(u)\} \\ \psi(u) &= c\left\{\exp\left[-\alpha\left(\sqrt{-2iu+\beta^2}-\beta\right)\right]-1\right\}\end{aligned} \tag{1}$$

Figure7 shows a sample path of $\{Z_t\}$.

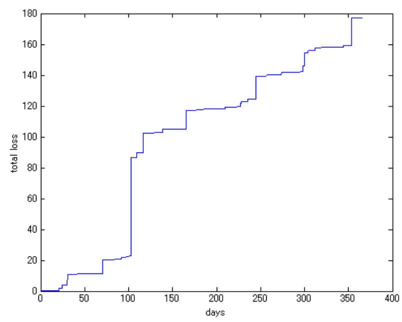


Fig. 7 A Sample Path of IG Type Compound Poisson Process

$(\alpha = 0.3868, \beta = 0.18076, c = 0.2005)$

If the parameters estimated by the classical method of moments are stated as $\alpha^{(\mathrm{CMM})}, \beta^{(\mathrm{CMM})}, c^{(\mathrm{CMM})}$, simple calculations provide the following equations:

$$
\begin{aligned}
\alpha^{(\mathrm{CMM})} &= \hat{h}_1 \frac{\beta^{(\mathrm{CMM})}}{\alpha^{(\mathrm{CMM})}} \\
\beta^{(\mathrm{CMM})} &= \sqrt{\frac{2}{-\hat{h}_2/\hat{h}_1 \pm \sqrt{-3\left(\hat{h}_2/\hat{h}_1\right)^2 + 4\hat{h}_3/\hat{h}_1}}} \\
c^{(\mathrm{CMM})} &= \frac{\hat{h}_1^{\,2}}{\hat{h}_2 - \hat{h}_1/\beta^{(\mathrm{CMM})}}
\end{aligned}
$$

Here, $\alpha^{(\mathrm{CMM})}, \beta^{(\mathrm{CMM})}, c^{(\mathrm{CMM})} > 0$

5.2 Γ (Gamma) Type Compound Poisson Process

ρ on $\mathbb{R}$ follows a Γ distribution, when

$$\rho(B) = \frac{\alpha^{\beta}}{\Gamma(\beta)} \int_B x^{\beta-1} \exp(-x\alpha) 1_{\{x>0\}}\, dx$$

However, $\alpha, \beta > 0$

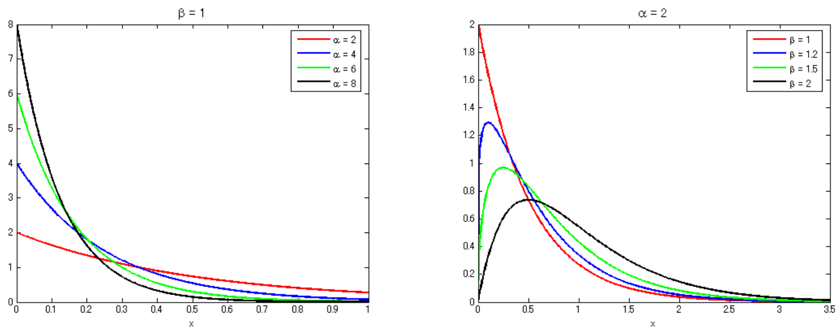


Fig. 8 Density Function of Γ Distribution

The characteristic function of Γ Distribution, $\hat{\rho}(u)$, follows :

$$\hat{\rho}(u) = \exp\left(1 - i\alpha^{-1}u\right)^{-\beta}$$

Therefore, Lévy Process, $\{Z_t\}$, that follows the Γ type compound Poisson process whose Lévy measure is:

$$\nu(dx) = \frac{c\alpha^{\beta}}{\Gamma(\beta)} x^{\beta-1} \exp(-x\alpha) 1_{\{x>0\}}\, dx$$

Also,

$$\psi(u) = c\left\{\left(1 - \frac{iu}{\alpha}\right)^{-\beta} - 1\right\}$$

Figure 9 shows a sample path of $\{Z_t\}$.

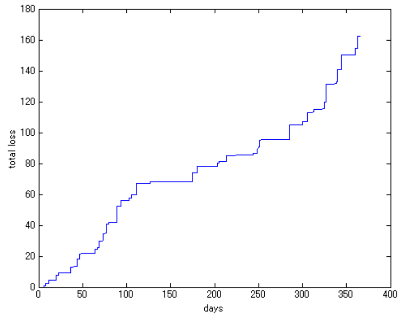


Fig. 9 A Sample Path of Γ Type Compound Poisson Process

$(\alpha = 0.22559, \beta = 0.48261, c = 0.20054)$

If the parameters estimated by the classical method of moments are stated as $\alpha^{(\mathrm{CMM})}, \beta^{(\mathrm{CMM})}, c^{(\mathrm{CMM})}$, simple calculations provide the following equations:

$$\alpha^{(\mathrm{CMM})} = \frac{-\hat{h}_1\hat{h}_2}{\hat{h}_2^2 - \hat{h}_1\hat{h}_3}$$
$$\beta^{(\mathrm{CMM})} = \frac{-\hat{h}_2^2}{\hat{h}_2^2 - \hat{h}_1\hat{h}_3} - 1$$
$$c^{(\mathrm{CMM})} = \frac{\hat{h}_1^2\hat{h}_2}{2\hat{h}_2^2 - \hat{h}_1\hat{h}_3}$$

Here, $\alpha^{(\mathrm{CMM})}, \beta^{(\mathrm{CMM})}, c^{(\mathrm{CMM})} > 0$

5.3 Lognormal Type Compound Poisson Process

ρ on $\mathbb{R}$ follows a lognormal distribution when

$$\rho(B) = \frac{1}{\sqrt{2\pi v}} \int_B \frac{\exp\left[-(\log x - m)^2/2v\right]}{x} 1_{\{x>0\}}\, dx$$

However,$v > 0$. In this, $\log(X) \sim N(m, v)$.

Therefore, Lévy process, $\{Z_t\}$, follows the Lognormal type compound Poisson process whose Lévy measure is:

$$\nu(dx) = \frac{c}{\sqrt{2\pi v}x} \exp\left[-\frac{(\log x - m)^2}{2v}\right] 1_{\{x>0\}}\, dx$$

Also, kth moment of the lognormal distribution, $m_k \left(:= \mathsf{E}[X^k]\right)$ follows the equation of

$$m_k = \exp\left(km + \frac{1}{2}k^2 v\right)$$

On the other hand, it is easy to see that

$$\psi^{(1)}(0) = c\hat{\rho}^{(1)}(0) = icm_1$$
$$\psi^{(2)}(0) = c\hat{\rho}^{(2)}(0) = -cm_2$$
$$\psi^{(3)}(0) = c\hat{\rho}^{(3)}(0) = -icm_3$$

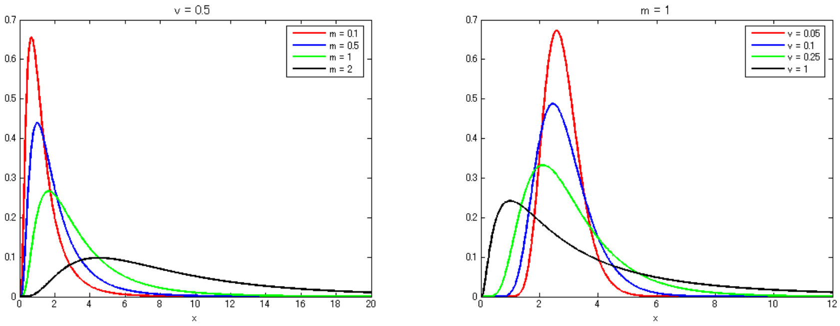


Fig. 10 Density Function of Lognormal Distribution

Simple calculations provide the following estimators by the classical method of moments:

$$
\begin{aligned}
m^{(\mathrm{CMM})} &= \log\left(\hat{h}_2/\hat{h}_1\right) - \frac{3}{2}\hat{v} \\
v^{(\mathrm{CMM})} &= \log\frac{\hat{h}_1\hat{h}_3}{\hat{h}_2^{\,2}} \\
c^{(\mathrm{CMM})} &= \exp\left[\log(\hat{h}_1)\left(\hat{m} + \frac{1}{2}\hat{v}\right)\right]
\end{aligned}
$$

Here, $\hat{v} > 0$

Figure 11 shows a sample path of $\{Z_t\}$.

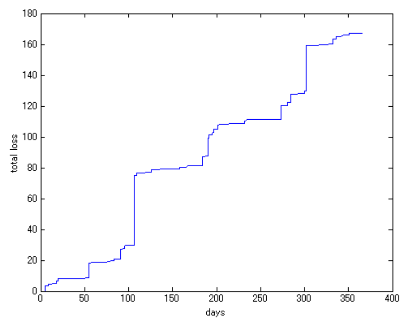


Fig. 11 A Sample Path of Lognormal Type Compound Poisson Process

$(m = -0.56274, v = 2.9082, c = 0.20057)$

5.4 Pareto Type Compound Poisson Process

ρ on $\mathbb{R}$ follows a Pareto distribution when

$$
\rho(B) = \int_B \frac{\alpha\beta^\alpha}{x^{\alpha+1}} 1_{\{x>\beta\}}\, dx, \quad \alpha, \beta > 0
$$

Therefore, Lévy process, $\{Z_t\}$, follows the Pareto type compound Poisson process whose Lévy measure is:

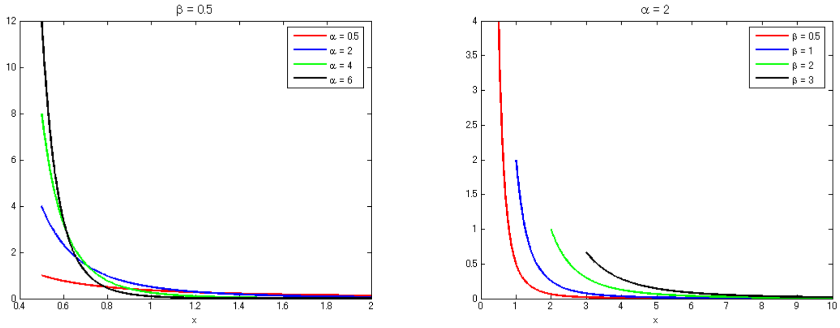


Fig. 12 Density Function of Pareto Distribution

$$\nu(dx) = c\frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}1_{\{x>\beta\}}\,dx$$

In order to generate a sample path following the Pareto type compound Poisson process, random numbers following a pareto distribution are generated by a method of reverse function:

$$Pareto(\alpha,\beta) \sim \beta\left(\frac{1}{1-\mathrm{Uniform}(0,1)}\right)^{1/\alpha}.$$

Figure 13 shows a sample path of Z_t.

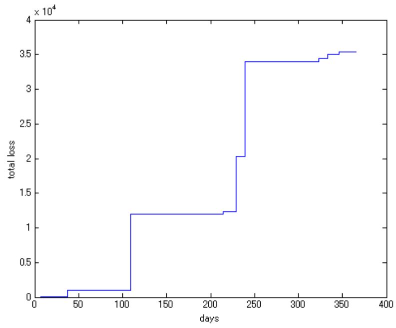


Fig. 13 A Sample Path of Pareto Type Compound Poisson Process

$(\alpha = 0.25019, \beta = 0.01039, c = 0.20053)$

Also,kth moment of Pareto distribution follows an equation of

$$m_k = \frac{\alpha\beta^k}{\alpha-k}, \quad k < \alpha$$

Therefore, simple calculations provide the following estimated parameters of,$\{Z_1\}$, by the classical method of moments:

$$
\begin{aligned}
\alpha^{(\mathrm{CMM})} &= 2 \pm \frac{\sqrt{A^2 - A}}{A - 1}, \qquad A := \frac{\hat{h}_1 \hat{h}_3}{\hat{h}_2^2} \\
\beta^{(\mathrm{CMM})} &= \frac{\hat{h}_2}{\hat{h}_1} \frac{\alpha^{(\mathrm{CMM})} - 2}{\alpha^{(\mathrm{CMM})} - 1} \\
c^{(\mathrm{CMM})} &= \hat{h}_1 \frac{\alpha^{(\mathrm{CMM})} - 1}{\alpha^{(\mathrm{CMM})} \beta^{(\mathrm{CMM})}}
\end{aligned}
$$

5.5 Weibull Type Compound Poisson Process

ρ on $\mathbb{R}$ follows a Weibull distribution when

$$
\rho(B) = \int_B \frac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left(-(x/\beta)^\alpha\right) 1_{\{x>0\}}\, dx, \qquad \alpha, \beta > 0
$$

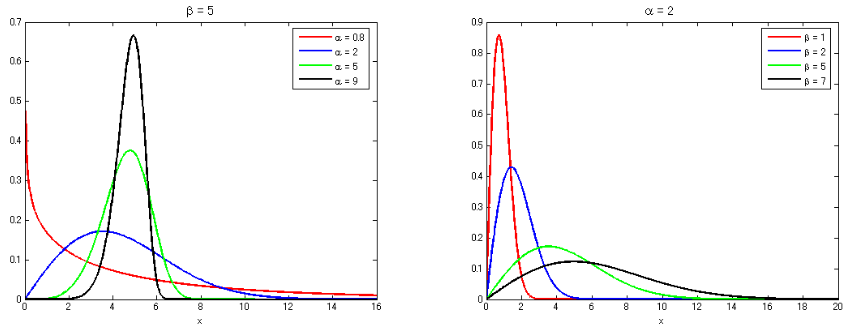


Fig. 14 Density Function of Weibull Distribution

Therefore, Lévy process, $\{Z_t\}$, follows the Weibull type compound Poisson process whose Lévy measure is:

$$
\nu(dx) = c \frac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right) 1_{\{x>0\}}\, dx
$$

Figure 15 shows a sample path of Weibull type compound Poisson process. Random numbers following a Weinbull distribution are generated by a method of reverse function:

$$
Weibull(\alpha, \beta) \sim \beta \left\{-\log\left[\mathrm{Uniform}(0,1)\right]\right\}^{1/\alpha}
$$

Also, kth moment of the Weibull follows:

$$
m_k = \beta^k \Gamma\left(1 + \frac{k}{\alpha}\right)
$$

If the parameters estimated by the classical method of moments are stated as $\alpha^{(\mathrm{CMM})}, \beta^{(\mathrm{CMM})}, c^{(\mathrm{CMM})}$, simple calculations provide the following results:

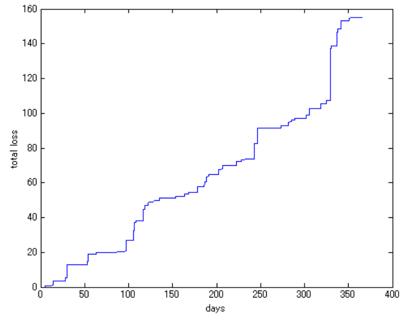


Fig. 15 A Sample Path of Weibull Type Compound Poisson Process

$(\alpha = 0.60858, \beta = 1.3321, c = 0.20055)$

$$
\begin{aligned}
\alpha^{(\mathrm{CMM})} &\in \left\{ \alpha \in \Re^+ \;\middle|\; \frac{\hat{h}_3}{\hat{h}_1} \frac{\Gamma(1+1/\alpha)}{\Gamma(1+3/\alpha)} - \left(\frac{\hat{h}_2}{\hat{h}_1}\right)^2 \left(\frac{\Gamma(1+1/\alpha)}{\Gamma(1+2/\alpha)}\right)^2 = 0 \right\} \\
\beta^{(\mathrm{CMM})} &= \pm \sqrt{\frac{\hat{h}_3}{\hat{h}_1} \frac{\Gamma(1+1/\alpha^{(\mathrm{CMM})})}{\Gamma(1+3/\alpha^{(\mathrm{CMM})})}} \\
c^{(\mathrm{CMM})} &= \frac{\hat{h}_1}{\beta^{(\mathrm{CMM})}\Gamma(1+1/\alpha^{(\mathrm{CMM})})}
\end{aligned}
$$

6 Methodology

6.1 Parameter Estimation

Since the focused models have characteristics of timely homogeneous, the time series $\{Z_i - Z_{i-1},\ i = 1, \cdots, n\}$ has actual values of $\xi = \{\xi_i,\ i = 1, \cdots, n\}$ and they are samples of Z_1. Therefore, the parameters of Z_1 are estimated using these samples of ξ.

The frequency of loss occurrence is more or less once a day. This means that the compound Poisson process has a intensity parameter of less than one. Thus, as far as the daily data is concerned, the distribution of jump width can be separately estimated from the distribution of time for jump occurrence in the model estimation（See Appendix A).

The paper attempts to take the parameter estimation procedure as follows:

1. Parameters of Z_1 are estimated by the classical method of moments.

2. If the estimated intensity is much less than one, the distribution of jump width is estimated separately from the distribution of time for jump occurrence.

3. If the estimated intensity is larger than one, the estimated parameters by the classical method of moments are used for the model.

6.2 Model Selection Based on AIC

As previously stated, the model process is considered a combination of two separate models: a distribution of jump width and a distribution of time for jump occurrence. Accordingly, the distribution of time for jumping is the same factor among these focused models. The goodness-of-fit of those models is, in turn, the goodness-of-fit of the model distribution of jump width to the actual distribution of loss amounts.

With the daily data from 1992 to 1996, the parameters of the focuse five distinct models are estimated. Then, the best fitted model to the data is chosen based on the AIC criterion of a distribution of jump width.

7 Result, Implications and Limitations

The result is summarized in Table 1. The table shows that the Lognormal type compound Poisson process is considered the best fit to the risk among those focused models. The same result is visually estimated from Figure16-20.

Unfortunately, appropriate parameters of the Weibull type compound Poisson process cannot be obtained by the classical method of moments.

Conversely, the Pareto type compound Poisson process is considered the worst fit to the risk. The reason can be explained by the fact that a parameter of Pareto distribution, β, is greatly influenced by the minimum values of the sample data and another parameter, α, is the only parameter that is rather free from those minimum values.

7.1 Kolmogorov-Smirnov Test

Kolmogorov-Smirnov Test is conducted to examine the goodness-of-fit of the model to the data. However, since it is hard to calculate the expected frequency from the distribution function, Z_1, the expected frequency is estimated in the iterative procedure with 100,000 iterations.

As a result of the test, the P-Value of the Lognormal type compound Poisson process is 0.5588. It is implied that the model and the actual data are ”not significantly different”. Also, it is visually considered that Figure21 is very similar to the actual data.

The P-Value of each model is summarized in Table1. It is interesting to note that the Γ type compound Poisson process can be concluded ”not appropriate” to the model based on its P-Value, even though it is determined as the second best to the Lognormal type and is better than IG type compound Poisson process based on AIC.

The Weibull type compound Poisson process is considered comparatively good to the model based on its AIC of jump width distribution while it is rather worse than IG type compound Poisson according to Figure 22.

These results illustrate that the Γ distribution, the Pareto distribution and the Weibull distribution with the obtained parameters, whose density functions provide larger values as x closes to 0, are possibly ”inappropriate” to the model even though they are considered comparatively good based on their AIC of jump width.

On the other hand, IG distribution and Lognormal distribution, which density functions provide smaller values as x closes to 0, can be candidate to the model.

If one is considered approporiate to the model, it is then examined whether or not its jump width distribution model appropriately fit the actual jump width distribution. Among these candidate models as the loss models, Lognormal type compound Poisson process is considered better than IG type since Lognormal type is better fit the jump width distribution. In conclusion, the Lognormal type compound Poisson process is accepted as the model describing the U.S. product liability risk.

7.2 Limitations

Since the model is selected based on the goodness-of-fit partially, namely on comparison of distributions of jump width, the selected model is not necessarily at most the best for the risk. Also, the model is based on the loss data of a particular company. It is not necessarily concluded that the model is applicable to other corporations or the industry in general.

Further, the model is built based on the assumption that the U.S. product liability exposure is constant during those five year period. However, in reality, the exposure changes as time passes. For example, changes in the units of sales, the technology, the safety features, the legal environment, the medical costs and monetary values might affect the exposure.

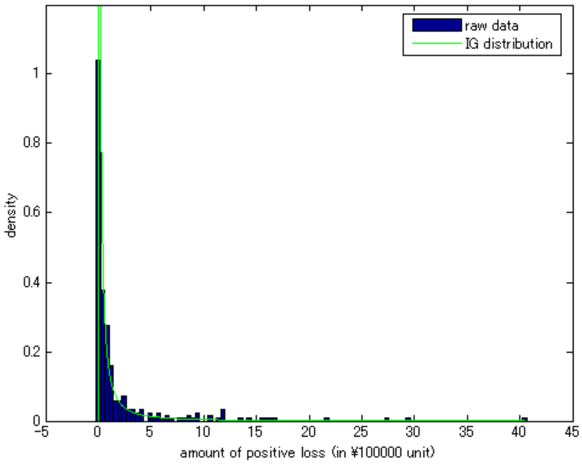


Fig. 16 Density Function of Jump Width

Raw Data and IG Distribution (with Maximum Likelihood Estimators)

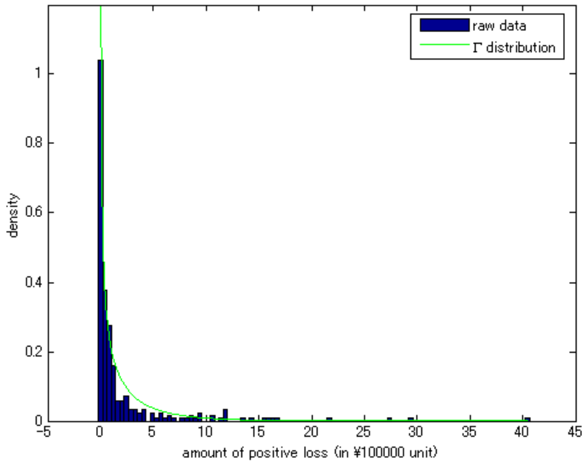


Fig. 17 Density Function of Jump Width

Raw Data and Γ Distribution (with Maximum Likelihood Estimators)

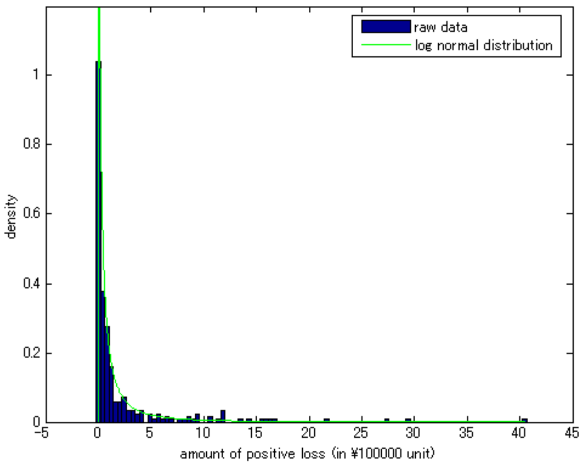


Fig. 18 Density Function of Jump Width

Raw Data and Lognormal Distribution (with Maximum Likelihood Estimators)

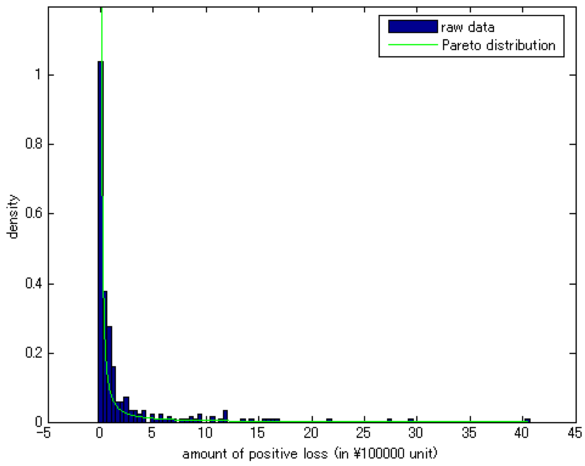


Fig. 19 Density Function of Jump Width

Raw Data and Pareto Distribution (with Maximum Likelihood Estimators)

Table 1 Estimation Result

IG type compound Poisson Process		α	β	c	AIC of Jump Width distribution	P-Value
	CMM	3.4574	0.3931	0.0405	(23425.56)	
	MLE(Sep.)	0.3868	0.1808	0.2005	930.982	0.4167
Γ type compound Poisson Process		α	β	c	AIC of Jump Width distribution	
	CMM	0.0984	0.1157	0.3027	(1269.483)	
	MLE(Sep.)	0.2256	0.4826	0.2005	926.564	0.0221
Lognormal type compound Poisson Process		m	v	c	AIC of Jump Width distribution	
	CMM	1.4683	0.6400	0.0595	(3426.297)	
	MLE(Sep.)	-0.5627	2.9082	0.2006	849.093	0.5588
Pareto type compound Poisson Process		α	β	c	AIC of Jump Width distribution	
	CMM	3.4546	6.7188	0.0376	(412611.9)	
	MLE(Sep.)	0.2502	0.0104	0.2005	1586.94	0.0184
Weibull type compound Poisson Process		α	β	c	AIC of Jump Width distribution	
	CMM	N/A	N/A	N/A	N/A	
	MLE(Sep.)	0.6086	1.3321	0.2006	886.124	0.1766

Note: Unfortunately, appropriate parameters of the Weibull type compound Poisson process cannot be obtained by the classical method of moments. With several trial initial values, the maximum likelihood estimators are obtained, of which the parameters providing the maximum likelihood are chosen for the model comparison.

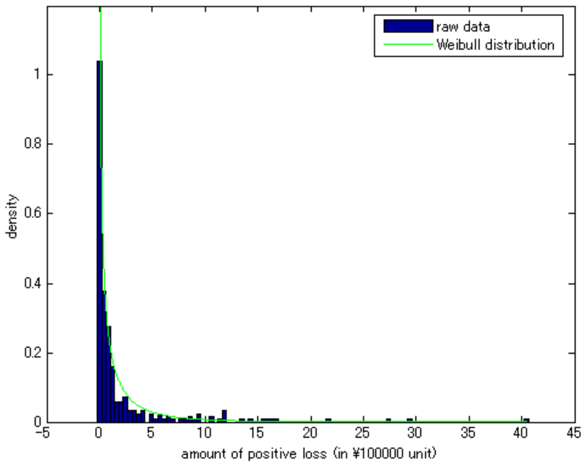


Fig. 20 Density Function of Jump Width

Raw Data and Weibull Distribution (with Maximum Likelihood Estimators)

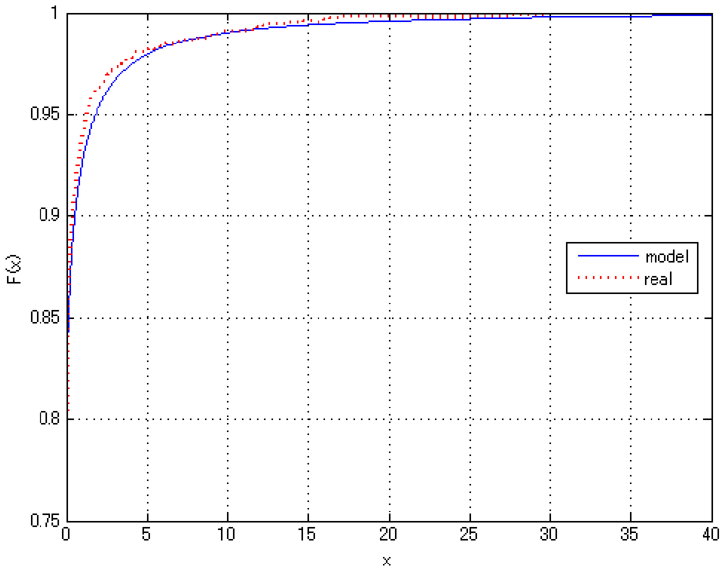
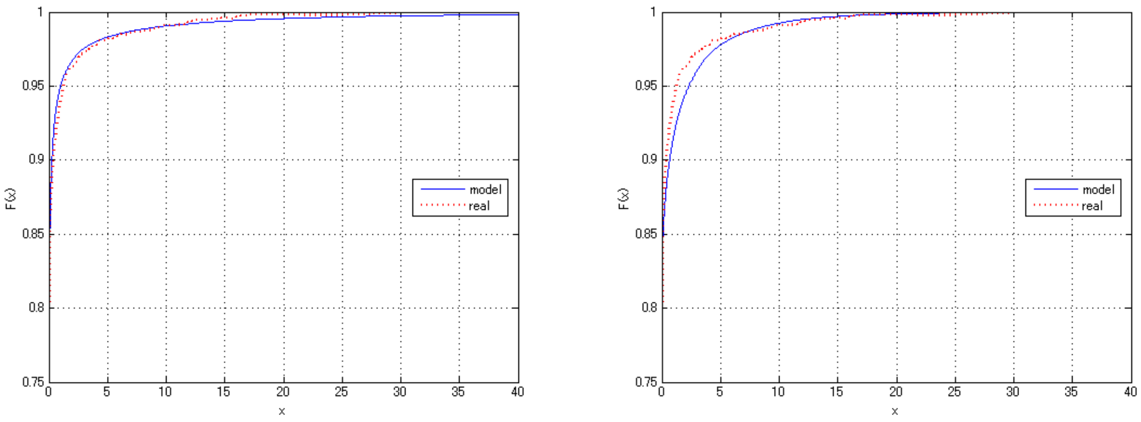


Fig. 21 Empirical Distribution Function and Cumulative Distribution Function of Z_1 following Lognormal type compound Poisson process



(a) IG Distribution

(b) Weibull Distribution

Fig. 22 Empirical Distribution Function and Cumulative Distribution Function of Z_1

8 Concluding Remarks

This paper determined the Lognormal type compound Poisson process as the good model to describe the risk.

Now being back to the original purpose of the study, the model is built for purpose of a feasibility study in establishing a captive. With the model, it is now possible to examine the cumulative distribution at a year end which is then used for the proforma statements in the feasibility study. Figure 23 illustrates the cumulative distribution of losses at the year end as a result of 10,000 iterations in Monte Carlo simulations. If the risk manager of Company A tends to view risks at the 90% confidence level, the blue portion of the figure illustrates that confidence level. On the other hand, the red portion shows 10% as the threshold exceedence. From the accumulated losses at the 90% confidence level, the initial capital and premium of the captive can be numerically obtained from simulations of proforma financial statements, which invites further research following this study.

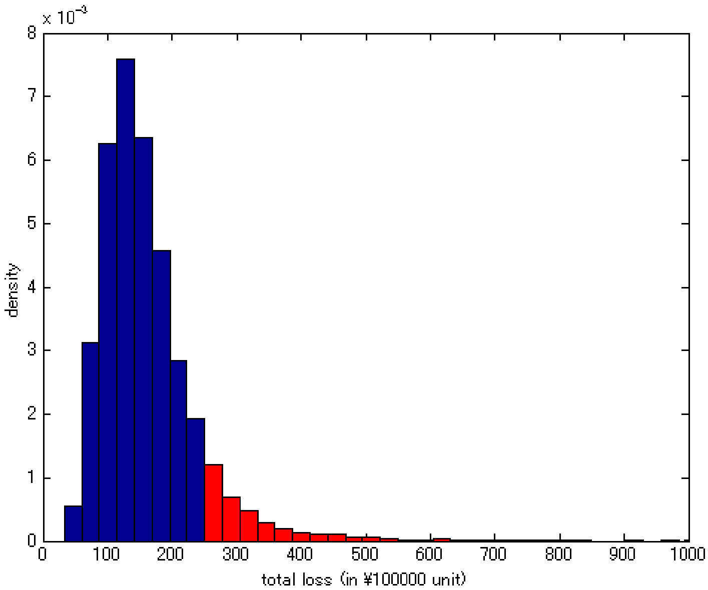


Fig. 23 Cumulative Loss Distributions at A Year End of Lognromal Type Compound Poisson Process

Appendix A

As Table 1 illustrates, the simulation results are analyzed when the intensity is estimated much smaller than one from the sample data. In such a case, one might wonder whether or not separate estimation of distributions for time for jumping and for jump width is appropriate. If yes, one might conclude that the estimated parameters in this method are appropriate to describe the actual model.

To examine the adequacy of this separating estimation method, parameters are reversely estimated from samples generated from the model with initial parameters. With 1,800 random samples generated in the method, the maximum likelihood estimation method provides numerical estimation of parameters when initial values are set at three times as much as the previously estimated parameters obtained by the classical method of moments.

The result of the estimated parameters are summarized in Table2. Since the estiamted parameters in this method provide more or less the same values among three sample data, it is concluded that the method is adequate when the model intensity is much less than one.

Table 2 Simulation Result

IG type compound Poisson process	$\alpha = 0.3868$	$\beta = 0.1808$	$c = 0.2005$
Sample Data 1	0.39079	0.18756	0.2240
Sample Data 2	0.40336	0.19303	0.2054
Sample Data 3	0.42627	0.18926	0.2065
Γ type compound Poisson process	$\alpha = 0.2256$	$\beta = 0.4826$	$c = 0.2005$
Sample Data 1	0.17746	0.48826	0.22749
Sample Data 2	0.22084	0.52728	0.23560
Sample Data 3	0.24000	0.47249	0.22843
Lognormal type compound Poisson process	$m = -0.5627$	$v = 2.9082$	$c = 0.2006$
Sample Data 1	-0.35129	2.9093	0.21733
Sample Data 2	-0.52186	2.8961	0.22197
Sample Data 3	-0.48414	2.8346	0.2255
Pareto type compound Poisson process	$\alpha = 0.2502$	$\beta = 0.0104$	$c = 0.2005$
Sample Data 1	0.23784	0.010675	0.22383
Sample Data 2	0.26650	0.010431	0.24087
Sample Data 3	0.24130	0.010484	0.20028
Weibull type compound Poisson process	$\alpha = 0.6086$	$\beta = 1.3321$	$c = 0.2006$
Sample Data 1	0.60363	1.5058	0.24789
Sample Data 2	0.59288	1.4532	0.22797
Sample Data 3	0.61494	1.3893	0.20998

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